A Wave Decomposition Method for the Active Control of Sound Absorption: Theory and First Experimental Results

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Introduction

Well known ANC [1] methods are based on generating a zero pressure field driving an appropriate signal through a secondary source. In a similar way, active absorption methods drive a signal through a secondary source not in order to minimize the sound field in front of it but trying to absorb the reflected wave generated by a sound wave impinging against a surface.

Guicking *et al.* [2] worked with periodic signals in an absorption system using analog circuits . Later, Orduña-Bustamante *et al.* [3] implemented an adaptive system based on the impedance control in front of the secondary source to achieve the absorption .

Using a one-dimensional sound field with two superimposed plane waves, one generated by a primary source and its reflected resultant, the method proposed use a secondary source to cancel the reflected part adjusting the *gain* and *phase shift* with regard to the primary source. To achieve this minimization of the reflected part, the total sound field is decomposed into its incident and reflected waves. This precise identification of the reflected part allow to create a control procedure to obtain an optimal absorption.

A theoretical and experimental study of this method is presented. Equations for this setup are shown. Initially pure tone signals are used to find the optimal operation point of the system and later on band-limited noise is used to evaluate the efficiency of the system.

Sound field inside a Kundt's tube with lateral neck

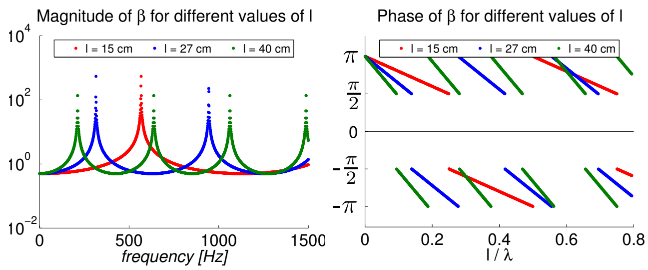
A Kundt's tube was used with two loudspeakers, the primary source in one of its extremes, a secondary source at the lateral neck and a rigid end in the other extreme as shown in Figure 2. The sound pressure field inside the tube can be expressed as

$$\begin{aligned} p(x) &= p_0\left[e^{-jkx} + \beta e^{-jkx} + (1+\beta)e^{-j2kl}e^{jkx}\right] \quad , x > 0 \\ p(x) &= p_0\left[e^{-jkx} + \beta e^{jkx} + (1+\beta)e^{-j2kl}e^{jkx}\right] \quad , x < 0 \end{aligned} \tag{1}$$

where k is the wave number, β is defined as the *complex gain* of the secondary source, which is at $x = 0$, and l is the distance between this source and the rigid end. Thus, for each equation, the first term corresponds to the sound pressure generated by the primary source, the next term corresponds to the sound contributed by the secondary source and the last term is contributed by the reflected wave of the rigid end. Since the idea is to produce real absorption through the secondary source, it is of interest to cancel all the reflected wave upstream to the neck, to obtain only a traveling wave from the primary to the secondary source (i.e. in the region $x < 0$). Investigating eq. (1) it can be seen, that

$$\beta + (1+\beta)\ e^{-j2kl} = 0, \tag{2}$$

is a necessary condition to cancel the reflected wave in the region $x < 0$. From eq (2), β can be drawn to show the values of magnitudes and phases where the secondary source acts as an active absorber (cf. Figure 1).



(a) **(b)**

Figure 1: (a) Magnitude of β for different values of l. (b) Phase of β for different values of l.

Wave decomposition

The general solution for a wave travelling in both directions inside a duct is given by

$$p(x) = \left[Ae^{-jkx} + Be^{jkx}\right] \tag{3}$$

where A and B are arbitrary complex magnitudes of the sound wave traveling in the positive and negative direction of x respectively. Eq. (3) allows a decomposition into its two parts. Measuring the sound pressure with two microphones separated by a distance Δx, A and B can be calculated as

$$A = \frac{p_2 - p_1 e^{jk\Delta x}}{e^{-jk\Delta x} - e^{jk\Delta x}} \tag{4}$$

$$B = -\frac{p_2 - p_1 e^{-jk\Delta x}}{e^{-jk\Delta x} - e^{jk\Delta x}}\ . \tag{5}$$

If p_1 and p_2 are sound pressure values in region $x < 0$, B describes the part of the wave, which is reflected towards the primary source. Therefore if B can be made to vanish, only a progressive wave traveling in the positive direction will remain and the reflected wave will be absorbed by the secondary source acting as an active absorber.

Test setup

Feeding the same signal into both sources but adding a phase shifter before the secondary loudspeaker it is possible to adjust the *gain* and *phase shift* of this source with regard to the primary. The adjustment of these parameters allows to reach the minimization of B. The test setup is shown in Figure 2.

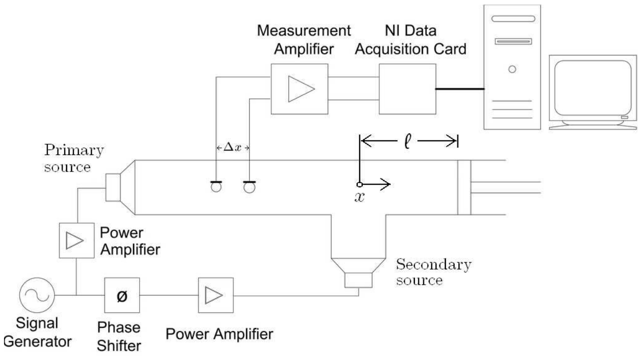


Figure 2: Measurement setup. Δx is the distance between microphones and l is the distance between the secondary source and the rigid end.

The signals are acquired from the microphones through a data acquisition card to a PC running a Matlab script developed for this measurement. As a first step pure tone signals were used to find the optimal absorption point for a given frequency, and later band-limited noise was fed into the system, leaving the parameters of *gain* and *phase shift* fixed. Several distances of Δx and l were tested.

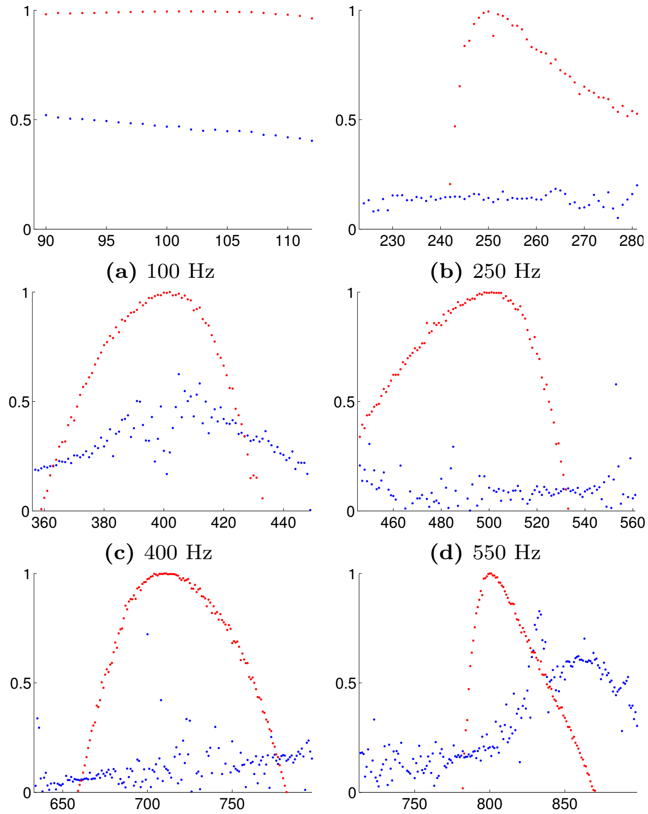
Results

To evaluate the performance of the system, the absorption coefficient α was measured using the two microphones technic (DIN EN ISO 10534-1). Preliminary measurements made with pure tone signals show high efficiency of the system, reaching almost in every case $\alpha \approx 1$ (cf. Table 1).

Table 1: Absorption coefficient α measured using pure tones, ($\Delta x = 10$ and $l = 15$).

f [Hz]	α	f [Hz]	α
100	1	700	1
250	0.99975	850	0.99999
400	1	1000	0.99999
550	0.99999	1250	0.99997

As a second step, curves of absorption coefficient were measured to evaluate the response of the system in a wider frequency range using band-limited noise. Some examples of curves measured are shown in Figure 3. The bandwidth [4] (BW) was used to compare the size of the lobe for these curves. The BW shown significant differences among curves of the same frequency, the only exceptional case was 100 Hz with a very similar value (≈ 83.7). Since the only parameter changed was l, the dissimilar decay pattern of each curve and consequently the different BW´s obtained, makes evident the influence of this parameter over the system response.



(a) 100 Hz
(b) 250 Hz
(c) 400 Hz
(d) 550 Hz
(e) 710 Hz
(f) 800 Hz

Figure 3: Absorption coefficient α versus frequency, using 1/3 octave band noise. System tuned for each frequency indicated. Red: System in optimal absorption condition, Blue: Control switched off, ($\Delta x = 10$ and $l = 15$).

Conclusions

In this work an active absorption system was presented. It was proven the well functioning of the methodology of wave decomposition to separate the total sound field in two waves traveling in opposite directions inside the tube. The high effectivity of the system using pure tone signals was shown, in almost every case measured the absorption coefficient reached was $\alpha \approx 1$. Curves of absorption coefficient were measured for different tuning frequencies. At the tuning frequency, each curve shows a good performance ($\alpha \approx 1$).

References

[1] Colin H. Hansen & Scott D. Snyder, *Active Control of Noise and Vibration.* E & FN Spon, (1997).

[2] D. Guicking, K. Karcher and M. Rollwage , *"Coherent active methods for applications in room acoustics."* J. Acoust. Soc. Am. **78**(4), 1426-1434 (1985).

[3] F. Orduña-Bustamante and P. A. Nelson , *"An adaptive controller for the active absorption of sound."* J. Acoust. Soc. Am. **91**(5), 2740-2747 (1992).

[4] Michael Möser , *Ingeniería Acústica, Teoría y Aplicaciones*, (2004).